Managing The Agency Problem in Organisational Behaviour: A Multi-Level Optimisation and Incentive Compatibility Approach

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ABSTRACT

Organisational behaviour research has become an advanced area of corporate finance studies. Firms are faced with controversies of decision making between the board of directors (BOD) representing the shareholders being the principals and the managers being the agents, leading to principal agent problems. To deal with such problems, a significant element in research in organisational behaviour and finance has been dedicated to developing efficient organisational behaviour mechanisms which are able to serve the interests of the BOD (representing the owners or shareholders).

This paper argues that a principal-agent organisational behaviour mechanism of the principal agent system can be appropriately represented by implementing a multi-level optimisation and incentive compatibility model. Integrated multi-level modelling and incentive compatibility approach in organisational behaviour is a new development, even though seminal literature in multi-level modelling and incentive compatibility approach in organisational behaviour in the form of agency theory does exist. Therefore, further development of such literature is required.

This paper presents: (1) a critical literature survey of the issues of organisational behaviour and the contributions made by the present paper over the existing models of agency theory in organisational behaviour and finance; (2) a multi-level programming model of organisational behaviour and incentive compatibility mechanism; (3) an institutional mechanism which can address issues such as moral hazard, asymmetric information, incomplete contract, adverse selection, etc. in organisational behaviour; (4) some numerical and non-numerical examples of such a multi-level programming approach to organisational behaviour and their incentive and agency implications; and (5) methods for applied large scale modelling.

This study reveals that financial incentives of organisational behaviour such as variations in financial numeration being used, may not be enough to solve agency problems in firms. Development of corporate culture based on business ethics can be supplementary to the existing methods of organisational behaviour. This paper argues that the multi-level optimisation approach provides a useful modelling methodology for, and insights into, the agency problem in organisational behaviour.

JEL classification: G3, C6
Keywords: Organisational behaviour; agency theory; mathematical modelling
1. INTRODUCTION

Organisational behaviour research in economics is characterised by studies of a two-level decision system between the board of directors (hereinafter called “BOD”) being the principals representing the owners/shareholders, and the managers being the agents. Such studies have advanced into the studies of equilibrium resource allocation mechanisms. As different types of decision makers exist in an organisation, a multi-level general equilibrium model should be appropriate to represent the underlying multi-level decision making system.

Seminal literature in multi-level planning (at economy-wide, sectoral, or corporate level) for efficient resource allocation in a decentralization framework has developed over time (for surveys see Islam 1998; Fox, Sengupta, and Thorbecke 1973), following the earlier works on this subject by Arrow and Hurwicz (1960), Malinvaud (1967) and Dantzig and Wolfe (1961). An integrated multi-level modelling approach to multi-level planning has been developed recently (see Candler and Norton 1977; Islam 1998). Further development of the literature in the area of multi-level modelling of organisational behaviour is required; this type of modelling is not known to the present author (Heinrich 2002; Brandimarte 2002).

The objectives of this paper are: 1) to develop a multi-level optimisation model for organisational behaviour to appropriately represent the underlying decision making system; and 2) to use the aforementioned model for developing efficient organisational behaviour mechanisms.

The structure of this paper is as follows. Section 2 presents a review of organisational behaviour issues. Section 3 reports a critical literature survey of the modelling methodology for the principal agent system and the contributions made by the present paper. A multi-level programming model is defined in Section 4, followed by

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1This paper is partly based on Islam (1998), published by Springer Verlag. The author thanks Springer Verlag for their permission to use copyright materials in preparing this paper. It is a substantially revised version of a paper prepared for the Asia Pacific Economics and Business Conference, 2 – 4 October, 2002, Kuching, Malaysia. The author is grateful for the extensive research assistance provided by Amerta Mardjono in preparing this paper.
some numerical and non-numerical examples of multi-level programming for organisational behaviour in Section 5. Discussion of different institutional mechanisms that exist for organisational behaviour and the implications of multi-level optimisation models for building mechanisms for organisational behaviour is presented in Section 6. Discussion of the possibility for real life organisational behaviour large scale applications, and extensions of multi-level optimisation models and computational methods is reported in Section 7. The conclusions of the paper are summarised in Section 8.

2. ORGANISATIONAL BEHAVIOUR: THE ISSUES

The evolution of management studies has derived several phenomenal school of thoughts (Daft 2004) such as (1) scientific management, (2) organisation theory and (3) decision theory. Being an important aspect of management studies, organisational theory or behaviour has an initial history which is dated back to 2250 B.C., when Hammurabi released 282 laws governing among others business dealings, personal behaviour, interpersonal relations, wages, punishments, and a host of other societal matters (Wren 2005). In the context of modern management era, Robbins and Mukerji (1994) define organisational behaviour as ‘the actions of people at work’ (p. 234), with ultimate goals to explain and predict behaviour.

During its development, in line with the increasing complication of business dealings and firms’ characteristics, the growing aspect of organisational behaviour approach to management is the existence of agency problems (Wiesner and Millett 2001). In this context, Williamson (1999) discloses that as the ownership and control in firms are often separated, an inquiry is needed to investigate whether such problems expose the corporations to organizational ramifications. In agency problems, conflicts of interests often occur between different stakeholders in the corporations, such as between the BOD (representing the shareholders), the managers and employees (representing the agents), the financiers, etc.

The modern corporate system has been increasingly characterised by the separation between ownership and control of corporations. Such a separation has also become an
important concern in corporate finance research. ‘One of the major concerns of corporate finance is the design of mechanisms for corporate control – in other words, how to make managers do what BOD/shareholders want’ (Allen and Gale 2001, p. 20). This is a crucial area of study in corporate finance, popularly known as the principal agent theory, which is developed on the assumption that a well-developed market for corporate controls does not exist, therefore resulting in market failures, non-existence of markets, moral hazard, asymmetric information, incomplete contract, adverse selection, etc. In this respect, a number of governance mechanisms have been advocated and practiced including monitoring by financial institutions, prudent market competition, executive compensation, debt, and developing a responsible board of directors, markets for corporate control and concentrated holdings.

Formal models to design efficient control mechanisms for organisational behaviour have been developed. Although for a multi-level decision system such as organisational behaviour, a multi-level model is necessary. Existing models which are primarily game theory models (see Allen and Gale 2001; Eatwell, Milgate and Newman 1987) are not fully integrated multi-level models. Moreover, some of these game theoretic principal agent models suggest that efficient corporate financial systems can be designed to overcome the agency problem in organisational behaviour. This view has been challenged by some researchers implying that mechanisms implied by game theoretic principal agent models for efficient organisational behaviour are not adequate for designing a desirable organisational behaviour system.

This paper argues that an integrated multi-level model is appropriate for analysing organisational behaviour, because a financial incentive system alone may not be enough to solve the agency problem in organisational behaviour.
3. A MODELLING METHODOLOGY

The issue of efficient allocation of resources leads to the development of a model based on the literature on multi-level decision making or decentralisation, irrespective of the nature of the organisation of the underlying economy.

At the initial stage of the development of the literature on multi-level decision making and decentralisation as in Arrow and Hurwicz (1960) and Malinvaud (1967), theories of decentralisation were developed rigorously with an emphasis on conceptual issues in specifying decentralisation processes of decision making. Subsequently the literature was extended by developing operational approaches for qualitative numerical development of multi-level decision models. Dantzig’s (Dantzig and Wolfe 1961) work was pioneering in this area.

In the multi-level approach of Dantzig, a decomposition decision rule for solving a large scale linear programme is developed; the inter-relations among the decomposed sub-systems are given a decentralisation interpretation in the sense that the decomposed sub-systems were the decentralised decision making units of the whole system. The decentralised decision making modelling was formalised and developed further by Kornai (Kornai and Liptak 1965; Goreux and Manne 1973) as two-level planning with separate different decision making problems at different levels.

Although those studies have contributed considerably to this field of knowledge, they do have limitations. Multi-level models have several limitations. Firstly, the whole planning problem is represented initially as a single level decision problem, which is then decomposed into different level decision problems. The single level mathematical model’s solution algorithm provides information for multi-level decision making. The multi-level decision problems are not explicitly specified by the single level planning model. These models have only type objective functions. Therefore, they cannot represent the different objectives of central and sectoral decision making (Fox, Sengupta and Thorbecke 1973, p. 533). Also these models cannot calculate optimal taxes and subsidies, reflecting the differences between the shadow prices of the central policy maker and the cost of production at the sectoral level, required by the centre to influence behaviour of agents at
other levels. In the multi-level planning model of Goreux and Manne (1973) different level objectives can be included, but these models are not fully integrated, and the results obtained from the individual models cannot be completely checked, so there may be inconsistencies in the model results.

Candler and Norton’s work (1977) on multi-level decision making termed as multi-level programming and the subsequent studies (see Islam 1998; Kumar 1991) following their work are remarkable. Multi-level programming is a recently developed mathematical programming method which represents the hierarchical decision making system in an integrated multi-level optimisation framework. It is argued that a multi-level programming model can adequately represent organisational behaviour mechanisms in terms of the existence of the principal and agent in the corporate environment. Candler and Norton (1977) have also demonstrated that numerical multi-level models can produce results which are efficient, in the sense that the multi-level programming models determine a higher level of social welfare than that which is determined in a single level manager’s model of a sector.

Although applied work has developed well (Candler and Norton 1977; Islam 1998), the theoretical investigations using multi-level programming have so far been limited in the sense that these studies explore the theoretical implications of multi-level decision making in perfectly competitive financial market conditions only and they do not highlight the impact of convexity of the programming problem on decision making. These studies assume that the principal’s objective function or the objective function at the upper level is convex (for a maximization case), although nothing can be said about the convexity of the principal-agent set. There is a need for extending the method of multi-level programming to develop a generalised theory of multi-level financial system programming.

In the next section of this paper (which is partly based on Islam 1998), it is shown that multi-level programming which incorporates the theory of multi-level optimisation and decision making is an appropriate methodology for modelling hierarchical decision making under different financial market conditions. It is also argued that although non-convexity is a general problem of multi-level programming, it is possible to specify a non-convex/non-concave upper level objective function in a multi-level organisational
behaviour program and the multi-level program can generate useful propositions compared to the current literature.

This paper produces some analytical, as well as, numerical results from non-convex non-concave multi-level programs. This study shows the application of a multi-level financial market optimisation model in representing organisational behaviour mechanism and the policy implications of the model results for corporate organisation.

4. MULTI-LEVEL PRINCIPAL-AGENT SYSTEM AND MULTI-LEVEL PROGRAMMING

4.1 Multi-Level Principal-Agent System

In a two-level decision making system, the upper decision makers are the BOD representing the shareholders and the lower level decision makers are the managers being the agents. Both the BOD/shareholders and the managers engage in optimising behaviour for making decisions: the BOD/shareholders attempt to maximize their share earnings and the managers/agents optimise their criteria functions such as maximisation of profit, minimisation of cost, etc. Such a multi-level financial market decision making system requires a multi-level optimisation approach for system modelling. Multi-level programming is a methodology for such multi-level optimisation. Although it is very close to game theory in terms of strategic interactions, the methodologies of multi-level programming (mathematical structure, solutions algorithm and computer programs) are different from that of game theory. A general two-level optimisation model (i.e., a two-level programming model) appropriate for representing interactive hierarchical organisational behaviour system is discussed in the next section.

4.2 A Multi-Level Programming Model: Definition and Specification

A general multi-level organisational behaviour model may be specified as follows:

\[
\max_{\{x_1\}} W = W_1 x_1 + W_2 x_2 \quad \text{s.t. } A_1 x_1 + A_2 x_2 \leq B_i
\]

Principal’s Model
\[
\begin{align*}
\max_{x_2} \Pi &= P_2 x_2 \quad & s.t. \quad A_2 x_2 \leq B_2
\end{align*}
\]

Agent’s Model

where:

- \( W_1 \) and \( W_i = (N \times 1) \) and \( (M \times 1) \) vectors of policy weights;
- \( A_{1p} \) and \( A_{2p} = (K \times N) \) matrices of coefficients of the principal’s sub-model;
- \( A_2 = (L \times M) \) matrix of technological coefficients;
- \( P_1 = (1 \times M) \) vector of prices of outputs;
- \( X_1 = (N \times 1) \) vector of policy variables (such as financial incentives, general shareholders decisions about production and marketing activities);
- \( X_2 = (M \times 1) \) vector of agent’s variables; and
- \( B_1 \) and \( B_2 = (N \times 1) \) and \( (M \times 1) \) vectors of RHS parameters.

4.2.1 Variables

We first define the variables of a multi-level program. Target variables: the variables that represent the objectives of the BOD/shareholders. Instrument variables: the variables that are adopted by the BOD/shareholders as instruments to control and influence the system under consideration in order to achieve the targets. Agent’s variables: the variables that represent the corporate production decisions and technology.

4.2.2 Functions

A multi-level organisational behaviour market program, as in this case, contains two optimisation sub-problems: the principal’s optimisation problem and the agent’s optimisation problem. These two sub-problems again have two objective functions; the principal’s objective function and the agent’s objective function, as well as three types of opportunity sets. They are the principal’s opportunity set, the agent’s opportunity set and the principal-agent opportunity set.

The principal’s objective function represents the decision criteria of the BOD/shareholders, while the manager’s objective function represent the criteria of optimising behaviour of the managers being the agents. The principal’s opportunity set is a set of ranges of values of the policy instruments. The manager’s opportunity set represents
the feasible region of the manager’s variables in the system and the principal-agent opportunity set shows the range of the manager’s variables under a set of values of the policy instruments.

4.2.3 Optimal Decisions and Organisational behaviour

In the theory of economic choice, the optimal values of the activities are the decisions or policies that optimise the decision criteria or objective functions of the principal and agent. In the above multi-level optimisation framework (Model 1), the optimal values of the variables or decisions are those for which the objective functions of different level decisions (principals and agents) attain their optimal values simultaneously ($x_1^*$ and $x_2^*$).

5. MULTI-LEVEL PRINCIPAL-AGENT SYSTEM MODELS

In this section we provide some specific examples of multi-level organisational behaviour models and show how these models can be used to derive specific decisions of the principal and the agent.

5.1 Competitive Market: Numerical

First we present a numerical model and its decision implications. For relating our numerical example to the general definition of multi-level programming provided in (1), we make the following assumptions:

$$x_1 = 0, \ A_{1_p} = 0, \ A_{2_p} = 0, \ B_1 = 0$$

The multi-level program is specified now as (with equality constraints):

$$\max_{\{x_1, x_2\}} W = x_1 x_2$$

$$\max_{\{\Pi, x_1, x_2\}} \Pi = 4x_1 + 3x_2 - 2x_1^2 - 2x_2^2 - t_1x_1 - t_2x_2 \quad \text{s.t.} \quad 10x_1 + 4x_2 = 200$$
where:

\[ x_1, x_2 = \text{outputs}; \quad \text{and} \]
\[ t_1, t_2 = \text{taxes imposed on these outputs}. \]

The welfare function has been formulated so that it is neither convex nor concave. This has the policy implication that the BOD/shareholders are only interested in the reallocation of the given resources of the firm.

We have assumed a competitive market condition where the market prices of the outputs \( x_1 \) and \( x_2 \) are given (which are 4 and 3).

The manager’s objective function maximises the aggregated profit in the economy. The prices are given and are independent of the levels of output. The costs of the production increases as the level of output increases.

We assume that the supply of the primary resources is fixed and the total supplies of secondary resources are constrained by the total availability of primary resources.

### 5.1.1 Single Level BOD/Principal’s Model

In most real life modelling work, the manager’s models are specified as a single-level model, instead of a multi-level model, such as the following:

\[
\begin{align*}
\max_{\{x_1, x_2\}} & \quad W = x_1 x_2 \\
\text{s.t.} & \quad 10x_1 + 4x_2 = 200
\end{align*}
\]

and the results of the programming problem are used to formulate production policy.

The application of the model results for formulating optimal production policies are demonstrated below by solving the model as a constrained optimisation problem.

The Lagrangean of the above problem is:

\[
L = x_1 x_2 + \lambda (200 - 10x_1 - 4x_2)
\]
By differentiating partially the Lagrangean function and equating to zero, we get a set of simultaneous equations:

\[
\frac{\partial L}{\partial \lambda} = x_2 - 10\lambda = 0 \\
\frac{\partial L}{\partial x_1} = x_1 - 4\lambda = 0 \\
\frac{\partial L}{\partial x_2} = 200 - 10x_1 - 4x_2 = 0
\]

By solving the equations, we get:

\[
\begin{align*}
x_1 &= 10 \\
x_2 &= 25 \\
\lambda &= 2.5
\end{align*}
\]

The resource allocation and policy implications of the results are that the economy should produce 10 and 25 units of the two types of resources so that the principal’s objective function is maximised.

Most existing organisational behaviour models are single level models like the one presented above. This adoption of such a single-level model for organisational behaviour is a gross misrepresentation of the real life environment. To show how the results of the single-level model can be inappropriate to formulate policies, let us solve the multi-level organisational behaviour program.

5.1.2 Multi-Level Model

Firstly, we solve the manager’s sub-model of the multi-level corporate model. The Lagrangean of the manager’s sub-problem is:

\[
L = 4x_1 + 3x_2 - 2x_1^2 - 2x_2^2 - t_1x_1 - t_2x_2 + \lambda(200 - 10x_1 - 4x_2)
\]
By differentiating partially the Lagrangean function and putting them equal to zero, we get the following system of equations:

$$\frac{\partial L}{\partial x_1} = 4 - 4x_1 - t_1 - 10\lambda = 0$$
$$\frac{\partial L}{\partial x_2} = 3 - 4x_2 - t_2 - 4\lambda = 0$$
$$\frac{\partial L}{\partial \lambda} = 200 - 10x_1 - 4x_2 = 0$$

(6)

Solution of the above set of equations provides:

$$x_1 = (116 - 29t_1 - 1870 - 25t_1 - 10t_2) / 116$$
$$x_2 = (87 - 29t_2 - 748 - 10t_1 - 4t_2) / 116$$

(7)

If we substitute the values of $x_1$ and $x_2$ in the principal’s objective function and, differentiate partially with respects to $t_1$ and $t_2$ as:

$$\frac{\partial W}{\partial t_1} = x_1 \frac{\partial x_2}{\partial t_1} + x_2 \frac{\partial x_1}{\partial t_1} = 0$$
$$\frac{\partial W}{\partial t_2} = x_1 \frac{\partial x_2}{\partial t_2} + x_2 \frac{\partial x_1}{\partial t_2} = 0$$

(8)

we get the values of $t_1$ and $t_2$:

$$t_1 = -10.79$$
$$t_2 = -30.48$$

and the values of $x_1$ and $x_2$ as after substituting the values of $t_1$ and $t_2$ back to the
expressions for $x_1$ and $x_2$ in (8):

\begin{align}
(9) \quad x_1 &= 14.32 \\
&
\quad x_2 &= 12.27
\end{align}

The policy applications of the above results may now be analysed as follows. The principal adopts the financial remuneration policy of $t_1 = -10.79$ and $t_2 = -30.48$ and the agent produces $x_1 = 14.32$ and $x_2 = 12.27$ units of output. This is the optimal allocation of resources in a multi-level corporate framework.

**5.1.3 Comparisons of Multi-Level and Single-Level Models**

To compare the multi-level optimisation results with a single-level principal’s model results, the results of a single-level principal’s model are used to derive the optimal output levels. Then optimal financial increments and reductions are determined which are required to achieve these output levels by influencing the behaviour of the individual economic agents. The decision making of the agent is specified in the manager’s sub-model. From the manager’s model, the optimal conditions for the maximisation of profit are developed as the following:

\begin{align}
\frac{\partial \pi}{\partial x_1} &= 4 - 4x_1 - t_1 = 0 \\
\frac{\partial \pi}{\partial x_2} &= 3 - 4x_2 - t_2 = 0
\end{align}

If we substitute the value of $x_1$ and $x_2$ from (6) in (11) then we have:

\begin{align}
(11) \quad 4 - 4(10) - t_1 &= 0 \\
&
3 - 4(25) - t_2 &= 0
\end{align}
We get the values of $t_1$ and $t_2$ as:

(12) \[ t_1 = -36 \]
\[ t_2 = -97 \]

The values of the present financial remuneration instruments are higher than that of their values in the multi-level programming case. This suggests that if a single-level principal’s model is used to determine the output level and the financial remuneration policies, so that the optimal output levels that optimise the principal’s objective function is achieved, the firm adopts a relatively higher level of financial remuneration instruments, i.e. higher level of policy controls.

If we compare the values of the corporate welfare function in the cases of single and multi-level models, we can see that the corporate welfare level is lower in the multi-level model (175.71) compared to the single level model (250). A single level model may therefore overstate the potential corporate welfare of a organisational behaviour scheme, and thus misrepresents the welfare implications of an optimal organisational behaviour scheme.

5.2 A General Modelling Exercise

To generalise the above results, now we specify a general model of appropriate organisational behaviour. The multi-level organisational behaviour schemes:

\[
\begin{align*}
\max_{\{t_1, t_2\}} & \quad W = x_1 x_2 \\
\text{s.t.} & \quad \max_{\{s_1, s_2\}} \quad \Pi = p_1 x_1 - p_2 x_2 - c_1 x_1^2 - c_2 x_2^2 - t_1 x_1 - t_2 x_2 \\
& \quad \text{s.t.} \quad A_1 x_1 + A_2 x_2 = B
\end{align*}
\]

5.2.1 Single Level Model
Let us now consider the resource allocation and organisational behaviour implications of adopting a single-level model. The single-level principal’s model is:

\[
\begin{align*}
\max_{x_1, x_2} & \quad W = x_1 x_2 \\
\text{s.t.} & \quad A_1 x_1 + A_2 x_2 = B
\end{align*}
\]

The Lagrangean function of the problem is:

\[
L = x_1 x_2 + \lambda (A_1 x_1 + A_2 x_2 - B)
\]

By differentiating partially and putting equal to zero we get the following set of equations:

\[
\begin{align*}
\frac{\partial L}{\partial x_1} &= x_2 + A_1 \lambda = 0 \\
\frac{\partial L}{\partial x_2} &= x_1 + A_2 \lambda = 0 \\
\frac{\partial L}{\partial \lambda} &= A_1 x_1 + A_2 x_2 - B = 0
\end{align*}
\]

or:

\[
\begin{bmatrix}
0 & 1 & A_1 \\
1 & 0 & A_2 \\
A_1 & A_2 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
B
\end{bmatrix}
\]

Solving by using the Cramer's rule:
\[ x_1 = \frac{B}{2A_1} \]

\[ x_2 = \frac{B}{2A_2} \]

\[ \lambda = \frac{-B}{2A_1 A_2} \]

\( x_1, x_2 \) are the optimal output levels determined in the owner control model.

### 5.2.2 Multi-Level Model Solution

For finding the optimal values of \( x_1 \) and \( x_2 \) for any given \( t_1 \) and \( t_2 \), we differentiate partially the manager’s sub-problem's Lagrangean function and put it equal to:

\[ L = p_1 x_1 - p_2 x_2 - c_1 x_1^2 - c_2 x_2^2 - t_1 x_1 - t_2 x_2 + \lambda (A_1 x_1 + A_2 x_2 - B) \]

\[ \frac{\partial L}{\partial x_1} = p_1 - 2c_1 x_1 - t_1 + \lambda A_1 = 0 \]

\[ \frac{\partial L}{\partial x_2} = -p_2 - 2c_2 x_2 - t_2 + \lambda A_2 = 0 \]

\[ \frac{\partial L}{\partial \lambda} = A_1 x_1 + A_2 x_2 - B = 0 \]

For solving the set of linear simultaneous equations, we redefine them in matrix form:
By solving the set of simultaneous equations using Cramer's rule, we find:

\[
\begin{align*}
 x_1 & = \frac{A_2^2(p_1 - t_1) + A_1A_2(t_2 - p_2) + 2c_2A_1B}{2(c_1A_2^2 + c_2A_1^2)} \\
 x_2 & = \frac{2c_1A_2B + A_1[A_2(t_1 - p_1) - A_1(t_2 - p_2)]}{2(c_1A_2^2 + c_2A_1^2)}
\end{align*}
\]

(22)

Now we substitute the values of \( x_1 \) and \( x_2 \) in the principal's sub-problem in (14) to find the optimal values of \( t_1 \) and \( t_2 \).

Since:

\( W = x_1x_2, \)

we redefine:

\[
\begin{align*}
 \frac{\partial W}{\partial t_1} &= x_1 \frac{\partial x_2}{\partial t_1} + x_2 \frac{\partial x_1}{\partial t_1} = 0 \\
 \frac{\partial W}{\partial t_2} &= x_1 \frac{\partial x_2}{\partial t_2} + x_2 \frac{\partial x_1}{\partial t_2} = 0
\end{align*}
\]

(23)

and get:
To solve for $t_1$ and $t_2$ we have rearranged the two equations as:

$$
\begin{align*}
\begin{bmatrix}
2A_1^2A_2^2 & -2A_1^3A_2 \\
-2A_1A_2^3 & 2A_2^4A_2^2
\end{bmatrix}
\begin{bmatrix}
t_1 \\
t_2
\end{bmatrix}
&= 
\begin{bmatrix}
2A_1^2A_2^2 & -2A_1^3A_2 \\
-2A_1A_2^3 & 2A_2^4A_2^2
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\end{align*}
$$

(25)

If we define the right-hand side expressions as $F_1$ and $F_2$ and solve the two simultaneous equations, we get:

$$
\begin{align*}
t_1 &= \frac{2A_1A_2^2F_1 + F_2(A_1^3A_2 + A_1A_2^3)}{2A_1^4A_2^4 + 2A_1^3A_2^5 - 2A_1^4A_2^2 - 2A_1^3A_2^6} \\
t_2 &= \frac{2A_1A_2^2F_1 + F_2(A_1^3A_2 + A_1A_2^3)}{2A_1^4A_2^4 + 2A_1^3A_2^5 - 2A_1^4A_2^2 - 2A_1^3A_2^6}
\end{align*}
$$

(26)

In both the cases, the values of the denominators are zero, because the determinant of the coefficient matrix is zero. So we cannot find any unique solution to the principal’s sub-model.

Let us explore the restrictions that may help us find a solution to the multi-level organisational behaviour problem. Since the determinant of the coefficient matrix is zero,
the first possible restrictions would be on the technological coefficients. But as the indeterminacy is arising in case of the principal’s sub-optimisation problem, we can do so by the restrictions on the financial remuneration instruments, $t_1$ and $t_2$.

One simple restriction which solves the problem is that either one of $t_1$ and $t_2$ is zero. Then we can find the values of $t_1$ and $t_2$.

The possible policy implication of this result is that a multi-level financial market programming model provides a remuneration policy package of either adopting $t_1$ and $t_2$ for effective organisational behaviour. And if the objective of the managers is to maximise profit and this is consistent with the objectives of the BOD/shareholders, this can be done in our example by adopting only one financial policy ($t_1$ or $t_2$).

### 5.2.3 Comparison of Single-Level and Multi-Level Models

We have output levels determined from the policy level. The financial policy pursued to achieve the optimal output levels in a decentralised economy by influencing the decisions of the economic agents may be determined as shown below. In our example the decision criterion is profit. So any government financial policy will affect profit earned by the firms. We need to determine the levels of taxes and subsidies that will induce the firms to produce the amounts of goods that were optimal in the central control policy model.

Following the steps adopted in the previous section, we now compare the results of the single-level and multi-level models. The model results will provide the levels of financial remuneration (increases or reductions) that will induce the manager to achieve the principal’s ideal resource allocation.

To determine such optimal $t_1$, $t_2$ values we first differentiate partially the profit function and set it equal to 0.

The profit function with the tax and subsidy is:

$$
\Pi = p_1 x_1 + p_2 x_2 - c_1 x_1^2 - c_2 x_2^2 - t_1 x_1 - t_2 x_2
$$

(27)

We set the partial derivatives equal to 0 as:
\[
\frac{\partial \Pi}{\partial x_1} = p_1 - 2c_1x_1 - t_1 = 0 \\
\frac{\partial \Pi}{\partial x_2} = p_2 - 2c_2x_2 - t_2 = 0
\]  

(28)

If we substitute the optimal values of \( x_1 \) and \( x_2 \) derived from the principal’s sub-model (equation (16)) in the optimal conditions for profit maximisation, we get the following values of \( t_1 \) and \( t_2 \) that will maximise profit for the firm and achieve the principal’s optimal output targets:

\[
t_1 = \frac{p_1 - c_1B}{A_1} \\
t_2 = \frac{p_2 - c_2B}{A_2}
\]  

(29)

The optimal values of \( t_1 \) and \( t_2 \) depend on the per unit profit of the per unit resource uses where the costs are evaluated only in terms of cost of each of output (\( c_1 \) and \( c_2 \)).

Similarly as in the case of the previous numerical example, if we compare the results of the multi-level financial market program and the single-level program, we can conclude that the multi-level programming model provides a policy solution with less shareholder control.

6. INSTITUTIONAL MECHANISMS FOR ORGANISATIONAL BEHAVIOUR

A traditional multi-level optimisation model of organisational behaviour with financial incentives as non-convex and unique governance mechanism does not exist. This shows the limitations of the traditional financial remuneration system for providing incentives for organisational behaviour. Several alternative corporate mechanisms can be adopted to provide improved organisational behaviour systems. One of the approaches
which significantly differs from present practices is to realign the interests of the BOD/shareholders (the principals) and managers (the agents) to have a single objective for both (Akerlof 1970; Allen and Gale 2001). In this case, the organisational behaviour problem can be represented by a single-level model such as the models of cooperative game theory or a single level principal’s model as in (3); a unique optimal allocation of resources is possible to identify. This is possible through non-economic incentives including the introduction of a strong ethical business culture in organisational behaviour. Other possible mechanisms (Allen and Gale 2001) include concentrated owners and share governance.

7. MULTI-LEVEL PROGRAMMING APPLICATIONS: APPLIED PROBLEMS AND COMPUTATIONAL METHODS

In this section, we discuss the various possible forms of multi-level organisational behaviour models and how to develop and compute a real-life large scale organisational behaviour scale.

7.1 Different Forms of Models

In this paper, we have developed cost/price control multi-level programming models for organisational behaviour. Resource control multi-level programming models (through the control of the RHS) have also been developed in the existing literature. In a resource control multi-level programming, the efficient prices of inputs and outputs to be implemented in the economy are the shadow prices, while the optimal activities are the output targets of the sectors or industries suggested by the optimum solution of the program. The output targets, which are endogenously determined, are the controls available to attain the optimal values of activities and the shadow prices are efficient prices.

Although this paper has demonstrated the extensions of a multi-level modelling approach to the formulation of optimal financial incentives for for-profit firms, the method can also be applied to a non-profit firm. We know that a single-level optimisation model can be adopted to specify an efficient allocation of resources for a economic organisation.
irrespective of the underlying social organisation of the economy (Islam 2001; Burmeister and Dobell 1970). Similarly, a multi-level optimisation model can be appropriate for representing the specification of organisational behaviour and efficient allocation of resources in any type of the economy since there will be multiple decision makers with different degrees of autonomy in every type of economy.

7.2 Applied Modelling and Computational Methods

Real life large scale multi-level organisational behaviour models can be developed for formulating an optimal organisational behaviour mechanism. This type of model can be numerically implemented by the algorithm and corporate program discussed below.

7.2.1 Solution Algorithm

A multi-level principal agent organisational behaviour model can be numerically implemented as a multi-level program. A multi-level program is considered as a collection of nested optimisation problems at different levels (Candler and Norton 1977). As it is a recently developed mathematical programming technique, the main difficulty with a multi-level programming (MLP) model is that there are no satisfactory algorithms for its implementation. Many algorithms are based on some sort of transformation of the original problem. This makes the size of the transformed MLP large in relation with the original problem. Commercial computer programs (which can implement the existing algorithms) are not available.

To compute the MLP solution easily and efficiently through an alternative strategy which can overcome the above problems, the Parametric Programming Search (PPS) algorithm has been developed in Islam (1998). In this approach the lower level problem of a complete multi-level program is solved by the author as a parametric programme, and the alternative optimum (basic) solutions to the lower level problem are searched to find the one that optimises the upper level objective function and at the same time satisfies the constraints on the policy choices. Several validation criteria for testing the sustainability of the PPS algorithm were applied. Firstly, the results produced by the PPS algorithm were found to be close to the true optimum results. Other criteria in the form of the efficiency in
CPU time, and the cost and efficiency in the extension and transfer of the algorithm were applied and the PPS algorithm was found satisfactory.

Generally, the problems of existence, uniqueness and global optimality of multi-level programming are still not resolved. The properties of the feasible region generated by the interactive decisions of the decision makers are not known since they are not explicit, algebraically or numerically. The region may be empty, disjoint, and even non-convex. In the first two cases, there will be a solution existence problem; and in the third case, there is the problem of the determination of uniqueness and global optimum.

Accepted views regarding these problems of policy existence, uniqueness and global optimality may be summarised as follows (Islam 1998; Candler and Norton 1977). In a large-scale real world problem, the problem of policy existence may not be encountered and appropriate algorithms can be adopted to overcome the policy uniqueness problem and to find the global optimum of a multi-level programming policy model.

7.2.2 Computer Program

The computer program GAMS (Brooke, Kendrick, Meeraus and Raman 1998) can also be used conveniently to implement the PPS algorithm. The multi-solve facility in GAMS along with an exogenously specified parametric search given by a loop can implement the program. Any other optimisation packages with parametric programming options can also be adopted.

8. CONCLUSION: INCENTIVE COMPATIBILITY

A multi-level optimisation model such as multi-level programming is necessary to formalise the underlying two-level decision making system in organisational behaviour. However, it is difficult to determine the existence, uniqueness and global optimality of the solution to a multi-level organisational behaviour programming compared to a single-level mathematical programming. Given this issue, the imposition of suitable restrictions on the control variables can solve these problems in a multi-level program, even in a non-convex, non-concave, multi-level program case.
Multi-level modelling of organisational behaviour provides some improved understanding of the organisational behaviour mechanism and the role of corporate finance such as financial remuneration to managers in helping to design an efficient governance mechanism. In addition to financial incentives, substantial reorientation in corporate culture and behaviour based on business ethics resulting in an alignment and compatibility of the goals of the BOD/shareholders and the managers, etc. can be useful in designing an efficient organisational behaviour mechanism.
REFERENCES

